## Grade $10^{\text {th }}$ CBSE

## Mathematics: CIRCLES

Q1) Prove that the length of tangents drawn from an external point to a circle are equal.

Q2) Triangle $A B C$ is drawn to circumscribe a circle of radius 3 cm , such that side $B C$ is Tangent to circle meeting it at $D$ and length of $B D$ and $D C$ is 6 cm and 8 cm respectively. Find the length of side $A B$ and $A C$ if area of triangle is $63 \mathrm{~cm}^{2}$.


Q3) In given figure, $A B$ is a chord of a circle with centre $O$ and $A D$ is a tangent. If $\angle B A D=60^{\circ}$, find $\angle A C B$

Q4) If the angle between two tangents drawn from an external point $P$ to a circle of radius a and centre $O$ is $60^{\circ}$ then find the length of $O P$.

Q5) In a circle with centre $O$, a diameter $A B$ and a chord AD are drawn. Another circle is drawn with $A O$ as diameter to cut $A D$ at $C$.

Prove that: $\mathrm{BD}=2 \times \mathrm{OC}$


Q6)Prove that the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.

## Solutions

## Grade $10^{\text {th }}$ : Circles

Sol.1) To Prove AP = BP
Proof:
In triangle $\triangle \mathrm{AOP}$ and $\triangle \mathrm{BOP}$
$\angle O A P=\angle O B P=90^{\circ}$ [tangents subtend $90^{\circ}$ at the center]

$\mathrm{OA}=\mathrm{OB}$ [ radius of same circle]
$\mathrm{OP}=\mathrm{OP}$ [Common]
$\therefore \triangle \mathrm{AOP} \cong \triangle \mathrm{BOP}$
Hence AP = BP [by CPCT]
Hence proved.

Sol. 2)
$B D=B E=6 \mathrm{~cm} \quad$ [tangents to a circle from an exterior point]
II ${ }^{l y} C D=C F=8 \mathrm{~cm}$ and $A E=A F$
$\operatorname{ar}(\triangle \mathrm{ABC})=\operatorname{ar}(\triangle \mathrm{AOB})+\operatorname{ar}(\triangle \mathrm{AOC})+\operatorname{ar}(\triangle \mathrm{BOC})$

$\operatorname{ar}(\triangle \mathrm{AOB})=\frac{1}{2} O E \times A B=\frac{1}{2} \times O E \times(A E+E B)=\frac{1}{2} \times 3 \times(A E+6)$
$\operatorname{ar}(\triangle \mathrm{AOC})=\frac{1}{2} O F \times A C=\frac{1}{2} \times O F \times(A F+F C)=\frac{1}{2} \times 3 \times(A F+8)$
$\operatorname{ar}(\triangle \mathrm{BOC})=\frac{1}{2} O D \times B C=\frac{1}{2} \times O D \times(B D+D C)=\frac{1}{2} \times 3 \times(6+8)$
$\operatorname{ar}(\triangle \mathrm{ABC})=\operatorname{ar}(\triangle \mathrm{AOB})+\operatorname{ar}(\triangle \mathrm{AOC})+\operatorname{ar}(\Delta \mathrm{BOC})$
$\left.\operatorname{ar}(\Delta \mathrm{ABC})=\frac{1}{2} \times 3 \times(A E+6)+\frac{1}{2} \times 3 \times(A F+8)+\right)=\frac{1}{2} \times 3 \times(6+8)$
$A E=A F$
$\operatorname{ar}(\Delta \mathrm{ABC})=\frac{1}{2} \times 3(A E+6+A E+8+6+8)$
$63=\frac{1}{2} \times 3(2 \mathrm{AE}+28)$
$A E=7$

Sol. 4)

$\angle A B D=60^{\circ}$
$\therefore \angle A E B=60^{\circ}$ [alternate segment subtend equal angle]
Now, $\angle A E B+\angle \mathrm{ACB}=180^{\circ} \quad$ [ACBE is a cyclic quadrilateral]
$\therefore \angle \mathrm{ACB}=120^{\circ}$

Sol.5)
$\mathrm{OA}=\frac{1}{2} A B \Longrightarrow O$ is a mid point of $A B \quad \because A B$ is diameter.
$A C=\frac{1}{2} A D \quad \because \perp O C$ Bisect chord $A D$
In $\triangle A B D, O C=\frac{1}{2} B D$
$\because$ Line joining the mid point of two sides of a triangle is half of the third side. Hence proved.

Sol. 6)
$R A$ is a diameter, hence $\angle R B A=90^{\circ}$
$\Rightarrow \angle A R B+\angle R A B=90^{\circ}$
Now, $\angle B A Q+\angle R A B=90^{\circ}$
$\because O A \perp$ tangent $P Q$
By (1) and (2) $\angle A R B=\angle B A Q$


Now $\angle A R B=\angle A C B$
[Angles of the same segment]
By (3) and (4)
$\angle A C B=\angle B A Q$
Hence proved.
Now,
$\angle B A P+\angle B A Q=180^{\circ}$ and $\angle A C B+\angle A D B=180^{\circ}$
$\therefore \angle B A P=\angle A D B$
Hence Proved.

