Grade 10th CBSE

Triangles - Similarity

Q1) A perpendicular drawn from the vertex of the right angle of a right-angled triangle divides the triangle into two triangles similar to each other and also to the original triangle. Prove it.

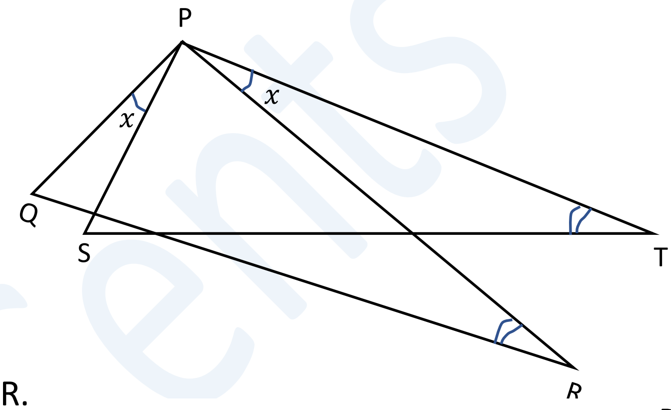
Q2) In given figure,

$\angle QPS = \angle RPT$ and

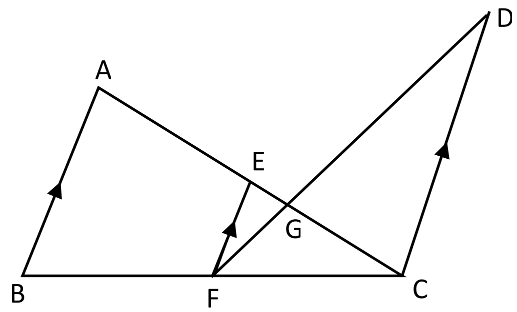
$\angle PRQ = \angle PTS$.

i) Prove that triangles PQR and PST are similar.

ii) If PT : ST =3: find ratio between QR:PR.



Q3) In given figure: AB$\parallel CD \parallel EF$. Given that AB = 7.5cm, EG = 2.5cm, CG =5cm and DC = 9cm. Calculate (i) EF (ii) AC



Q4) Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Q5) In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitude.

Q6) A street light bulb is fixed on a pole 6m above the level of the street. If a woman of height 1.5m cast a shadow of 3m, find how far she is away from the base of the pole?

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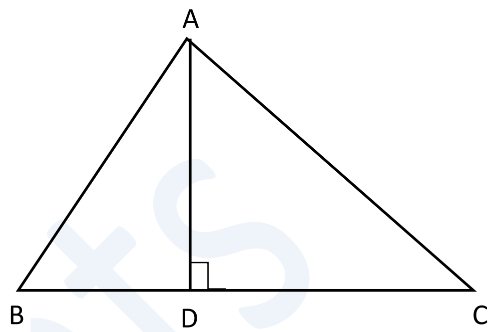
Solutions : Triangles - Similarity

Sol1) To prove $\Delta DBA \sim \Delta DAC \sim \Delta ABC$



Proof: In triangle ΔDBA and ΔABC

Statement	Reason
$\angle ABD = \angle ABC$	common
$\angle ADB = \angle CAB$	Each 90°
$\therefore \Delta DBA \sim \Delta ABC$	By AA Criterion

In triangle ΔDAC and ΔABC

Statement	Reason
$\angle ACD = \angle ABC$	common
$\angle ADC = \angle CAB$	Each 90°
$\therefore \Delta DAC \sim \Delta ABC$	By AA Criterion

Now,

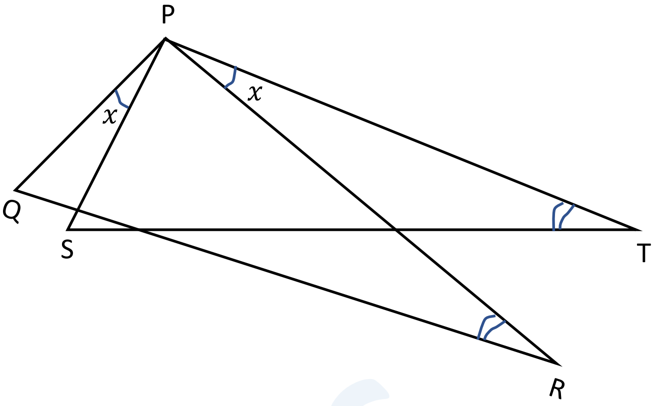
$\therefore \Delta DBA \sim \Delta DAC \sim \Delta ABC \quad [\because \Delta DBA \sim \Delta ABC \text{ and } \Delta DAC \sim \Delta ABC]$

Hence Proved.

Sol.2) i) To Prove $\Delta PQR \sim \Delta PST$

Proof: In triangle ΔPQR and ΔPST

Statement	Reason
$\angle QRP = \angle SPT$	Each angle equal to $\angle x + \angle SPR$
$\angle PRQ = \angle PTS$	Given
$\therefore \Delta PQR \sim \Delta PST$	By AA Criterion



Hence Proved.

ii) To prove QR : PR = 4 : 3

Proof:

Since corresponding sides f similar triangles are in proportion.

$\therefore \frac{QR}{ST} = \frac{PR}{PT} \Rightarrow \frac{QR}{PR} = \frac{ST}{PT} = \frac{4}{3}$ $\quad [\,Given, \frac{PT}{ST} = \frac{3}{4}\,]$

Hence QR : PR = 4 : 3

Hence Proved.

Sol.3) (i) In triangle ΔEGF and CGD.

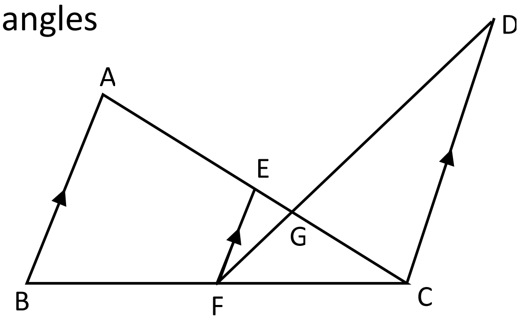
$\angle EGF = \angle CGD$ $\quad [vertically\ opposite\ angles]$

$\angle EFG = \angle FDC$ $\quad [Alternate\ interior\ angles]$

$\therefore \Delta EGF \sim \Delta CGD$ $\quad [by\ AA\ postulate]$

$\frac{EG}{CG} = \frac{EF}{CD}$ [corresponding sides of similar triangles are proportional]

$\frac{2.5}{5} = \frac{EF}{9}$ $\therefore EF = 4.5cm$



(ii)) In triangle ΔABC and EFC.

$\angle ABC = \angle EFC$ $\quad [corresponding\ angles]$

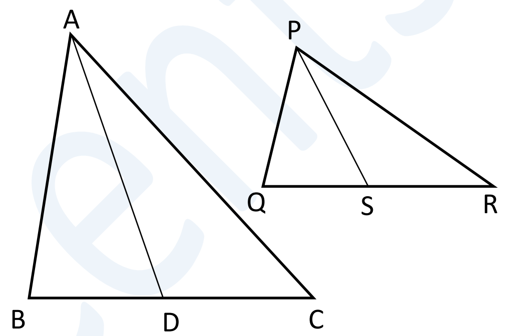
$\angle BAC = \angle FEC$ [*corresponding angles*]

$\therefore \Delta ABC \sim \Delta EFC$ [*by AA postulate*]

$\frac{AC}{EC} = \frac{AB}{EF}$ [corresponding sides of similar triangles are proportional]

$\frac{AC}{2.5+5} = \frac{7.5}{4.5} \therefore AC = 12.5cm$

Sol. 4) Draw AD and PS as medians of triangles ΔABC and ΔPQR



To prove

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$$

Proof:

Triangles $\Delta ABC \sim \Delta PQR$

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ....(1)

In triangle ΔABD and ΔPQS

Statement	Reason
$\angle B = \angle Q$	Corresponding angles of similar triangle.
$\frac{AB}{PQ} = \frac{BD}{QS}$	$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2BD}{2QS} = \frac{BD}{QS}$
$\therefore \Delta ABD \sim \Delta PQS$	By SAS Criterion

Hence $\frac{AB}{PQ} = \frac{AD}{PS}$ ....(2)

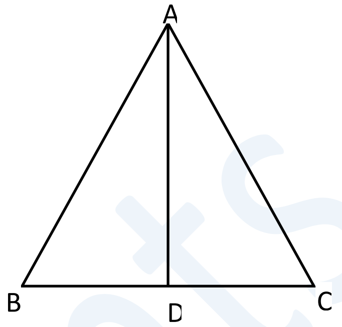
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{AD^2}{PS^2}$$

Hence Proved.

Sol.5) To prove $3AB^2 = 4AD^2$

Proof:

Let ΔABC is an equilateral triangle with AD as altitude, which bisect base BC at point D.



By Pythagoras Theorem,

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2$$

$$AB^2 = AD^2 + \frac{BC^2}{4}$$

$$AB^2 = AD^2 + \frac{AB^2}{4} \qquad [AB = BC]$$

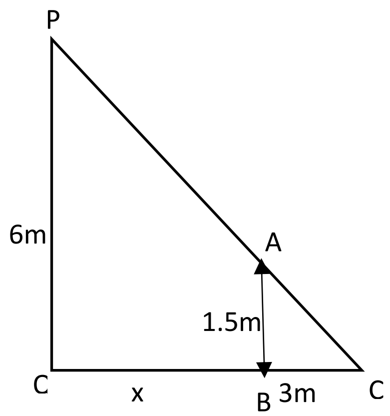
$$3AB^2 = 4AD^2$$

Hence Proved.

Sol.6)

In triangle ΔABC and ΔPQC

Statement	Reason
$\angle B = \angle Q$	90°
$\angle C = \angle C$	common
$\therefore \Delta ABC \sim \Delta PQC$	By AA Criterion



hence,

$$\frac{AB}{PQ} = \frac{CB}{CQ} = \frac{CB}{CB+QB}$$

$$\frac{1.5}{6} = \frac{3}{3+x} \qquad \Longrightarrow x = 9m$$