CBSE 10th

Arithmetic Progressions

Q1) How many whole numbers, each divisible by 9, lie between 200 and 500?

Q2)Which term of the A.P. 4, 11, 18, 25, ……… is 42 more than its 25th term?

Q3) Find the 12th term from the end in the A.P. 13, 18, 23….158.

Q4) If m times the mth term of an AP is equal to n times the nth term and m$\neq n$, show that its (m+n)th term is zero.

Q5) The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161. Find 30th term of this AP.

Q6) The ratio of the sums of first m and first n terms of an AP is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m-1):(2n-1)$.

Q7) If a, b and c are in A.P., show that (b + c), (c + a) and (a + b) are also in A.P.

Q8) In a school, students stand in rows. If 30 students stand in the first row, twenty-seven in the second row and 24 in the third row and six in the last row, find how many rows are there and what is the total number of students standing in the pattern?

Solutions

Arithmetic Progression

Q1) How many whole numbers, each divisible by 9, lie between 200 and 500?

Sol.1) First whole number in required AP is 207 and last one is 495.

$a = 207\,, d = 9\ and\ a_n = 495$

$a_n = \{a\ + (n-1)d\}$

$495 = \{207 + (n-1)9\}$

$$n - 1 = \frac{495 - 207}{9} = \frac{288}{9} = 32$$

There are 32 whole numbers each divisible by 9 lie between 200 and 500.

Q2)Which term of the A.P. 4, 11, 18, 25, ……… is 42 more than its 25th term?

Sol.2) For the given A.P.

a = 4 d = 7

$a_n = \{a\ + (n-1)d\}$

$a_{25} = \{4\ + (25-1)7\}$

$a_{25} = 172$

Required term is 42 more than its 25th term i.e.

$a_n = 172 + 42$ =214

$a_n = \{a\ + (n-1)d\}$

$214 = \{4\ + (n-1)7\}$

$$n - 1 = \frac{214 - 4}{7} = 30 \Longrightarrow n = 31$$

Required term is 31st term.

Q3) Find the 12th term from the end in the A.P. 13, 18, 23….158.

Sol. 3)

Let the number of terms in the given A.P. is n.

a = 13 d= 5

$a_n = \{a + (n-1)d\}$

158 = {13 + (n-1) 5}

$n - 1 = \frac{158-13}{5} = \frac{145}{5} = 29 \Longrightarrow n = 30$

12th term from end meaning (30-12+1 =19)th term from beginning.

$a_{19} = \{13 + (19-1)5\} = 103$

Q4) If m times the mth term of an AP is equal to n times the nth term and m$\neq n$, show that its (m+n)th term is zero.

Sol.4) Let a be the first term and d be the common difference of given A.P. Then,

$m.a_m = m.\{a + (m-1)d\}$

and

$n.a_n = n.\{a + (n-1)d\}$

Given

$m.\{a + (m-1)d\} = n.\{a + (n-1)d\}$

$\Rightarrow m.\{a + (m-1)d\}$ - $n.\{a + (n-1)d\}$=0

$\Longrightarrow m.a + m^2d - md$- $n.a - n^2d + nd = 0$

$(m-n).a + (m^2 - n^2)d + (n-m)d = 0$

$(m-n).\{a + (m+n-1)d\} = 0$

$m \neq n \therefore a + (m+n-1)d = 0$

Hence, $(m+n)^{th}$ term = 0

Q5) The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161. Find 30th term of this AP.

Sol5) $s_7 = 63$

$s_{14} = 63 + 161 = 224$

$s_7 = \frac{7}{2}\{2a + (7-1)d\} \Longrightarrow 7(a + 3d) = 63\ or\ a + 3d = 9 \quad \ldots\ldots(1)$

$s_{14} = \frac{14}{2}\{2a + (14-1)d\} \Longrightarrow 7(2a + 13d) = 224\ or\ 2a + 13d = 32\ldots(2)$

Solve (2) – 2x(1)

$7d = 14 \Longrightarrow d = 2, a = 3$

$a_{30} = \{a + 29d\} \Longrightarrow$ (3+29x2)=61

Q6) The ratio of the sums of first m and first n terms of an AP is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m-1) : (2n-1)$.

Sol6)

$s_m = \frac{m}{2}\{2a + (m-1)d\}$

$s_n = \frac{n}{2}\{2a + (n-1)d\}$

Given, $\frac{s_m}{s_n} = \frac{m^2}{n^2}$

$$\Longrightarrow \frac{\frac{m}{2}\{2a + (m-1)d\}}{\frac{n}{2}\{2a + (n-1)d\}} = \frac{m^2}{n^2}$$

$$\Longrightarrow \frac{n\{2a + (m-1)d\}}{m\{2a + (n-1)d\}} = 1$$

$\Longrightarrow \{2an + mnd - nd\} = 2am + mnd - md$

$\Longrightarrow 2a(n-m) - d(n-m) = 0$

$\Longrightarrow 2a - d = 0\ or\ 2a = d$

$$\frac{a_m}{a_n} = \frac{a + (m-1)d}{a + (n-1)d} = \frac{a + (m-1)2a}{a + (n-1)2a}$$

$$\Longrightarrow \frac{a(1 + 2m - 2)}{a(1 + 2n - 2)} = \frac{2m-1}{2n-1}$$

Hence proved.

Q7) If a, b and c are in A.P., show that (b + c), (c + a) and (a + b) are also in A.P.

Sol.7) a, b and c are in A.P

Then

a-(a+ b +c) , b-(a+ b +c) and c-(a+ b +c) will also be in A.P.

∵ (If same no subtracted from each term of an A.P. the resultant numbers are also in A.P.)

$\Longrightarrow -(b+c), -(a+c)\ and - (a+b) are\ also\ in\ A.P.$

$\Longrightarrow (b+c), (a+c)\ and (a+b) are\ also\ in\ A.P.$

Q8) In a school, students stand in rows. If 30 students stand in the first row, twenty-seven in the second row and 24 in the third row and six in the last row,

find how many rows are there and what is the total number of students standing in the arrangement?

Sol. 8) A.P. formed by the number of students standing in different rows

= 30, 27, 24, ……., 6

a = 30 , d = -3 and $l = a_n = 6$

$a_n = \{a + (n-1)d\}$

$6 = \{30 + (n-1)(-3)\}$

$\Rightarrow n = 9$

There are 9 rows in the arrangement of student.

Now

$$s_n = \frac{n(a+l)}{2}$$

$$s_n = \frac{9(30+6)}{2} = 162$$

There are 162 total students standing in the arrangement.