## CBSE Grade $9^{\text {th }}$

## Quadrilaterals

Q1) Prove that diagonals of parallelogram bisect each other.

Q2) In figure PQRS is a parallelogram, and $X, Y$ are the points on the diagonal QS such that SX = QY. Prove that

i) RXPY is a parallelogram.
ii) $\triangle P S X \cong \triangle R Q Y$
iii) $P X=R Y$ and $R X=P Y$

Q3) In figure, ABCD is a trapezium in which $\mathrm{AB} \| D C$ and $\mathrm{P}, \mathrm{Q}$ are the mid points of $A D$ and $B C$ respectively. $D Q$ and $A B$ when produced meet at $E$. Also $A C$ and PQ intersect at R. Prove that
(i) $\mathrm{DQ}=\mathrm{QE}$
(ii) $\mathrm{AB} \| P R$
(iii) $A R=R C$


Q4) E is the mid point of a median AD of $\triangle A B C$ and $B E$ is produced to meet AC at F . Show that $\mathrm{AF}=\frac{1}{3} A C$.

Q5)Prove that quadrilateral formed by internal angle bisector of a parallelogram is a rectangle.

Q6)In figure, $\triangle A B C$ is isoceles with $A B$ $=A C . D, E, F$ are the mid points of sides $B C, C A$, and $A B$ respectively. Show that the line segment $E F$ is bisected by $A D$.


## Solutions

## CBSE Grade $9^{\text {th }}$

## Quadrilaterals

Sol.1)
In $\triangle A O B$ and $\triangle C O D$, we have
$A B=C D$
$\angle O A B=\angle O C D$
and $\angle O B A=\angle O D C$
$\therefore \triangle A O B \cong \triangle C O D$


Hence, $O D=O B$ and $O A=O C$ (c.p.c.t)

Sol.2)
i) Join $R Q$, meeting $S Q$ at $O$.

Since diagonals of quadrilateral bisect each other $O S=O Q$ and $O P=O R$
Given SX = QY,
$\therefore O X=O Y$
$O P=O R$
Diagonals of quadrilateral PXRY, XY and PR bisect each other hence, PXRY is a parallelogram.
ii) In $\triangle P S X$ and $\triangle R Q Y$
$P S=R Q$ (opposite sides of parallelogram PQRS)
$P X=R Y$ (opposite sides of parallelogram PXRY)
$S X=Q Y$ (Given)

Hence $\triangle P S X \cong \triangle R Q Y$ By SSS
iii) $\mathrm{PX}=\mathrm{RY}$ (opposite sides of parallelogram PXRY)
$P Y=R X$ (opposite sides of parallelogram PXRY)

Sol.3) Given $A B \| D C$ and $P, Q$ are the mid points of $A D$ and $B C$ respectively. In $\triangle D C Q$ and $\triangle B E Q$
$C Q=B Q$
$\angle D Q C=\angle B Q E$ (vertically opposite angles)
$\angle D C Q=\angle B E Q$ (alternate interior angles)
$\therefore \triangle D C Q \cong \triangle B E Q$
$\therefore P Q \| A B$ (c.p.c.t)
i) In $\triangle A D E P$ is the mid point and $P Q \| A B$
$\therefore \mathrm{Q}$ is the mid point of DE (converse of mid point theorem)
Hence, DQ = QE
ii) $\mathrm{AB}\|P R \because A B\| P Q$ and $R$ lies on $P Q$
iii) In $\triangle A C B \mathrm{Q}$ is the mid point and $Q R \| A B$
$\therefore \mathrm{R}$ is the mid point of $A C$ (converse of mid point theorem)
Hence, $A R=R C$
Sol.4)
Draw DG || EF
In $\triangle A D G, E$ is mid point and $D G \| E F$
$\therefore F$ is mid point of $A G$, i.e $A F=F G$
In $\triangle F B C, D$ is mid point of $B C$, and $D G \| E F$
$\therefore G$ is mid point of $C F$, i.e $C G=F G$
Thus, $A F=F G=C G$


Hence $A F=\frac{1}{3} A C$

## Sol5)

$A B C D$ is a parallelogram. $A D \| B C$ and $A B \| D C$

$$
\angle D A B+\angle B A D=180^{\circ}
$$

$\because$ consicutive angles of parallelogram
$\therefore \angle D A F+\angle A D G=90^{\circ} \because D F$ and $D G$ are angle bisectors
$\therefore \angle A E D=90^{\circ} \because$ angle sum property .
$\therefore \angle G E F=90^{\circ} \because$ vertically opposite angles
Similarly, $\angle G H F=90^{\circ}, \angle H F E=90^{\circ}, \angle H G E=90^{\circ}$

## Sol6)

In $\triangle A B C, D$ and $E$ are mid points.
$\therefore D E \| B A$ and $D E=\frac{1}{2} B A=F A$
$\because$ mid point theorm
In $\triangle A F O$ and $\triangle D E O$

$D E=F A$
$\angle D E O=\angle A F O$ (Alternate angles)
$\angle E D O=\angle F A O$ (Alternate angles)
Hence, $\triangle A F O \cong \triangle D E O$ by ASA rule
Hence AO = DO by CPCT
Hence, AD bisects FE.

